Astronomy 10
Homework #9, Spring 2004
SOLUTIONS

Feel free to use the data in the book's appendices to answer these questions.

1. *This question is worth a total of five points. Two points for correctly identifying this as a SN Ia system and three points for actually calculating when this will occur.*

   You observe a double-line spectroscopic binary that consists of a $0.8M_\odot$ red giant and a $1.2M_\odot$ white dwarf. You also know that the red giant has filled its Roche lobe and is losing mass to the white dwarf at a rate of $2 \times 10^{-6} M_\odot$/yr. Something catastrophic will happen to the star within the next million years. Exactly when will this happen?

   Answer: Once the white dwarf has accreted enough mass from its companion so that it exceeds the Chandrasekhar Mass, **it will explode in a violent Type I Supernova.** The amount of mass it needs to gain before this occurs is $1.4 - 1.2\ M_\odot = 0.2\ M_\odot$. At a rate of $2 \times 10^{-6}\ M_\odot$/year, this will take $2 \times 10^{-1}\ M_\odot/(2 \times 10^{-6}\ M_\odot/\text{yr}) = \boxed{10^5\ \text{yr} = 100,000\ \text{years}}$. So the supernova will happen in 100,000 years.

2. *This question is worth a total of five points. Two points for the first two parts and one point for the last part.*

   Imagine the Sun were to collapse to a black hole (remember, this won't actually happen but, for fun, imagine it did).

   (a) How big would its Schwarzschild radius be?

   Answer: The Schwarzschild radius of the sun would be:

   $$R_S = \frac{2GM_\odot}{c^2} = \frac{2 \times 6.7 \times 10^{-11}\ \text{m}^3\ \text{kg}^{-1}\ \text{s}^{-2} \times 2 \times 10^{30}\ \text{kg}}{(3 \times 10^8\ \text{m/s})^2} = \boxed{3000\ \text{m}}. \quad (1)$$

   (b) What would happen to the orbits of the planets in our solar system?

   Answer: The orbits of the planets would remain unchanged since a black hole does not "suck." They will just continue to orbit the central mass, even if it is a black hole.

   (c) What is the ratio of the Schwarzschild radii of a $1M_\odot$ black hole and the supermassive black hole at the center of the Milky Way ($M = 2 \times 10^6 M_\odot$)?

   Answer:

   $$\frac{R_{S\odot}}{R_{S\text{mw}}} = \frac{\frac{2GM_\odot}{c^2}}{\frac{2GM_\text{mw}}{c^2}} = \frac{M_\odot}{M_\text{mw}} = \frac{1}{2 \times 10^6} = \boxed{5 \times 10^{-7}}. \quad (2)$$

3. *This question is worth a total of five points. Two points for the first part and three points for the last.*

   You observe a $2M_\odot$ pulsar with a pulsation frequency of 650 Hz and a radius of 10 km.

(a) What is the rotational speed of the pulsar at its equator in terms of the speed of light?

Answer: A pulsar with a pulsation frequency of 650 Hz rotates 650 times/second. At the equator one rotation is $2\pi r$ meters. So the equatorial speed is (650 Hz) $\cdot$ $(2\pi r)$, with a radius of 10 km. This gives a velocity of 4.084×10^7 m/s or $\boxed{0.14\text{c}}$.

(b) Some time later the frequency of the pulsar has decreased to 649 Hz. How much energy is radiated by the pulsar during this time? (Hint: Assume that the pulsar's kinetic energy in rotation is $\frac{1}{2}mv^2$, where m is the pulsar mass and v the equatorial velocity.)

Answer: The amount of energy radiated will be the difference in kinetic energy, or $(1/2)m(v_i)^2 - (1/2)m(v_f)^2$. The final velocity is found from the same equation as before $(f2\pi r)$, with 649 as the new frequency. Using $m = 2M_\odot$, we find the energy radiated to be $\boxed{1.03 \times 10^{43}\ \text{J}}$ (which is about the amount of energy the Sun radiates in one billion years!).

4. *This question is worth a total of five points. Three parts for the first part and two points for the last part.*

(a) For the same pulsar in problem 3, calculate the ratio of the centrifugal to gravitational force on the equator at the initial time. (Hint: The centrifugal force on a mass m is $F = mv^2/r$, where v and r are the equatorial velocity and radius respectively.)

Answer: The gravitational force is $F = GMm/r^2$ and the centrifugal force is what is given above. The ratio is then,

$$\frac{F_\text{C}}{F_\text{G}} = \frac{\frac{mv^2}{r}}{\frac{GMm}{r^2}} = \frac{v^2 r}{GM} = \boxed{0.063}. \tag{3}$$

(b) What would be the significance if this ratio were equal to or greater than one?

Answer: If this ratio were equal to (or greater than) one, the gravitational force would be just barely equal to (or less than) the centrifugal force. This would mean that gravity would no longer be able to hold this object together and it would have broken up (granted, the rotational velocity would have to be $0.54c$ for this to happen).

5. *This question is worth a total of five points. Three points for showing there work and how they balanced the two forces and solved for R and two points for the correct answer.*

How close can a binary system get to the super-massive black hole at the center of our Galaxy before it is ripped apart? Assume you have a binary system with two $1M_\odot$ stars that are separated by 1 AU. Calculate R, the distance in AU from the Galactic center at which the tidal force from the black hole is greater than the gravitational attraction between the two stars. (Note: The tidal force is,

$$F_\text{T} = \frac{2GM_\text{bh}m_*}{R^3} \cdot d, \tag{4}$$

where $M_{\rm bh}$ is the mass of the super-massive black hole (given in problem 2c), m_* is the mass of either star in the binary, and d is the separation of the two binary stars.)

Answer: We need to balance the tidal force and the gravitational attraction between the two binary stars,

$$F_{\rm T} = \frac{2GM_{\rm bh}m_*d}{R^3} = \frac{Gm_*m_*}{d^2}. \tag{5}$$

Solving for the distance from the black hole, R,

$$R = \left(\frac{2M_{\rm bh}d^3}{m_*}\right)^{\frac{1}{3}} = \boxed{160\ \text{AU}}. \tag{6}$$